13 Right-angled triangles

Syllabus reference

Measurement 4

- Right-angled triangles

In this chapter we will learn about right-angled triangles and how to apply your knowledge of them to problem solving.

ARE YOU READY?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillSHEET. Either click on the SkillSHEET icon next to the question on the *Maths Quest Preliminary Course* eBookPLUS or ask your teacher for a copy.

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Digital doc
SkillSHEET 13.1
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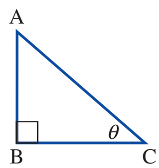
Labelling sides of a right-angled triangle

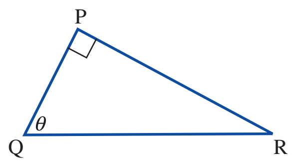
Labelling sides of a right-angled triangle

1 In each of the following, name the opposite side, adjacent side and the hypotenuse.

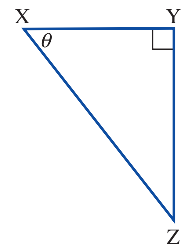
a



b



c



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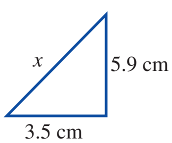
Using Pythagoras' theorem

Using Pythagoras' theorem

2 Find the value of x, correct to 1 decimal place.

a



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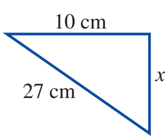

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SkillSHEET 13.3
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Rounding to a given number of decimal places

Rounding to a given number of decimal places

3 Round each of the following numbers to the number of decimal places indicated in brackets.

a 4.7368 [2] **b** 18.539 [1]
c 0.377 51 [3] **d** 507.182 09 [3]
e 10.797 [2] **f** 0.764 281 [4]

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Solving equations of the type $a = \frac{x}{b}$ to find x

Solving equations of the type $a = \frac{x}{b}$ to find x

4 Solve each of the following equations.

a $0.6 = \dfrac{x}{20}$ **b** $1.45 = \dfrac{x}{9}$

c $0.7328 = \dfrac{x}{4.7}$

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Digital doc
SkillSHEET 13.5
doc-1631
Solving equations of the type $a = \frac{b}{x}$ to find x

Solving equations of the type $a = \frac{b}{x}$ to find x

5 Solve each of the following equations.

a $0.25 = \dfrac{6}{x}$ **b** $7.5 = \dfrac{2.7}{x}$

c $0.1425 = \dfrac{76.95}{x}$

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SkillSHEET 13.7
doc-1634
Rounding angles to the nearest degree

Rounding angles to the nearest degree

6 Round each of the following to the nearest degree.

a 23.698° **b** 47.215°
c 27°24′34″ **d** 86°45′12″

13A Pythagoras' theorem

PYTHAGORAS OF SAMOS (circa 580BC–500BC)

During his life:

- Taoism is founded
- Kung-Fu-tse (Confucius) is born
- Buddhism is founded.

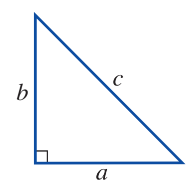
Pythagoras was a famous Greek mathematician and mystic who is now best known for his theorem about the sides of a triangle. He was born on Samos Island, near Greece. It is believed that he was born about 580 BC and died about 500 BC, but because of the way dates were recorded then, various dates are given for his life. Not much is known about his personal life, but we know that he had a wife, son and daughter.

When Pythagoras was a young man, he travelled to Egypt and Babylonia (Mesopotamia) where he learned much of his mathematics and developed an interest in investigating it further.

He founded a cult with the idea that 'the essence of all things is a number'. This group believed that all nature could be expressed in terms of numbers. They found, however, that some numbers could not be expressed as rational numbers, such as $\sqrt{2}$. They kept this information to themselves and there is a story that they killed one member who told somebody else about this problem.

Pythagoras showed that musical notes had a mathematical pattern. He stretched a string tightly and found that it produced a certain sound and then found that if he halved the length of the string, it produced a sound that was in harmony with the first. He also found that if it was not exactly half, or a multiple of a half, then it was a clashing sound. This approach is still used in musical instruments today.

He is credited with the discovery now known as 'Pythagoras' theorem'. This states that 'For a right-angled triangle, the square of the hypotenuse (long side) is equal to the sum of the squares of the two short sides' and is normally written as $c^2 = a^2 + b^2$. Other people knew of this idea long before he announced it, and there is a Babylonian tablet known as 'Plimpton 322' (believed to have been made about 1500 years before Pythagoras was born), which has a set of these sort of values (which are now called Pythagorean triples). He used a string with knots in it to demonstrate and make right angles.



Some examples of Pythagorean triples are:

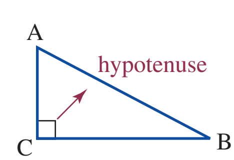
3, 4 and 5
8, 15 and 17

and a more difficult example: 20, 99 and 101.

Questions

1. Where was Pythagoras born?
2. Where did he travel to and learn most of his mathematics?
3. What is the formula for his famous theorem?
4. What did he start to investigate about patterns in nature?
5. What is the name of the ancient tablet that contains Pythagorean triples?
6. What is a Pythagorean triple?

Pythagoras' theorem allows us to calculate the length of a side of a right-angled triangle, if we know the lengths of the other two sides. Consider $\triangle ABC$ at right. AB is the **hypotenuse** (the longest side). It is opposite the right angle.

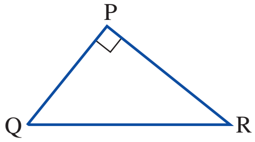


Note that the sides of a triangle can be named in either of two ways.

1. A side can be named by the two capital letters given to the vertices at each end. This is what has been done in the figure above to name the hypotenuse AB.
2. We can also name a side by using the lower-case letter of the opposite vertex. In the figure above, we could have named the hypotenuse 'c'.

WORKED EXAMPLE 1

Name the hypotenuse in the triangle at right.



THINK	WRITE
1 The hypotenuse is opposite the right angle.	
2 The vertices at each end or the lower case of the opposite vertex can be used to name the side.	The hypotenuse is QR or p.

Pythagoras' theorem

In any right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the two shorter sides. Writing this result as a formula we say:

$$c^2 = a^2 + b^2$$

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Interactivity
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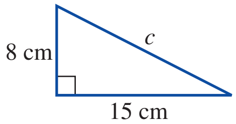
Pythagoras

This is the formula used to find the length of the hypotenuse in a right-angled triangle when we are given the lengths of the two shorter sides.

WORKED EXAMPLE 2

Find the length of the hypotenuse in the triangle at right.



THINK	WRITE
1 Write the formula.	$c^2 = a^2 + b^2$
2 Substitute the lengths of the shorter sides.	$c^2 = 8^2 + 15^2$
3 Evaluate the expression for c^2.	$= 64 + 225$ $= 289$
4 Find the value of c by taking the square root.	$c = \sqrt{289}$ $= 17$ cm

In this example, the answer was a whole number because we are able to find $\sqrt{289}$ exactly. In most examples this will not be possible. In such cases, we are asked to write the answer correct to a given number of decimal places or significant figures.

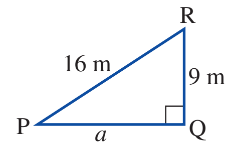
By rearranging Pythagoras' theorem, we can write the formula to find the length of a shorter side of a triangle.

If $c^2 = a^2 + b^2$ then $a^2 = c^2 - b^2$
and $b^2 = c^2 - a^2$.

The method of solving this type of question is the same as in the previous example, except we use subtraction instead of addition. For this reason, it is important to look at each question carefully to determine whether you are finding the length of the hypotenuse or one of the shorter sides.

WORKED EXAMPLE 3

Find the length of side PQ in triangle PQR, correct to 3 significant figures.



THINK	WRITE
1 Write the formula.	$a^2 = c^2 - b^2$
2 Substitute the lengths of the known sides.	$a^2 = 16^2 - 9^2$
3 Evaluate the expression.	$= 256 - 81$ $= 175$
4 Find the answer by finding the square root.	$a = \sqrt{175}$ $= 13.2$ m

Pythagoras' theorem states that in a right-angled triangle, $c^2 = a^2 + b^2$. The converse of this theorem states that if $c^2 = a^2 + b^2$ then the triangle is right angled. This is a method used by builders to ensure that a structure is 'square'.

WORKED EXAMPLE 4

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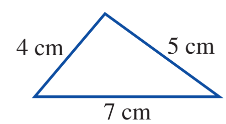
Worked example 4

Determine whether the triangle at right is right angled.

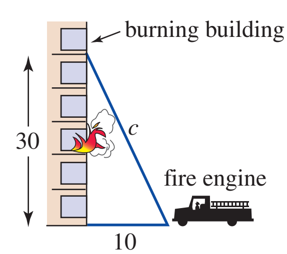


THINK	WRITE
1 Calculate c^2 and $a^2 + b^2$ separately.	$c^2 = 7^2$ $= 49$ $a^2 + b^2 = 5^2 + 4^2$ $= 25 + 16$ $= 41$
2 Write down an equality or inequality statement.	$c^2 \neq a^2 + b^2$
3 Write a conclusion.	Therefore the triangle is not right angled.

These formulas can be used to solve more practical problems. In these cases, it is necessary to draw a diagram that will help you to see which method for finding a solution is required. The diagram simply needs to represent the triangle; it does not need to show details of the situation described.

WORKED EXAMPLE 5

The fire brigade attends a blaze in a tall building. They need to rescue a person from the 6th floor of the building, which is 30 metres above ground level. Their ladder is 32 metres long and must be at least 10 metres from the foot of the building. Can the ladder be used to reach the people needing rescue?

THINK	WRITE
1 Draw a diagram and show all given information.	

2. Write the formula after deciding if you are finding the hypotenuse or a shorter side. — $c^2 = a^2 + b^2$
3. Substitute the lengths of the known sides. — $c^2 = 30^2 + 10^2$
4. Evaluate the expression. — $= 900 + 100$
   $= 1000$
5. Find the answer by taking the square root. — $c = \sqrt{1000}$
   ≈ 31.62 m
6. Give a written answer. — The ladder will be long enough to make the rescue.

REMEMBER

1. Make sure that you can identify the hypotenuse of a right-angled triangle. It is the side opposite the right angle.
2. If you are finding the length of the hypotenuse use $c^2 = a^2 + b^2$.
3. If you are finding the length of a shorter side use either $a^2 = c^2 - b^2$ or $b^2 = c^2 - a^2$.
4. Read the question carefully to make sure that you give the answer in the correct form.
5. Begin written problems with a diagram and finish with an answer written in words.

EXERCISE 13A Pythagoras' theorem

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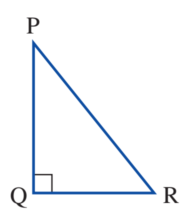
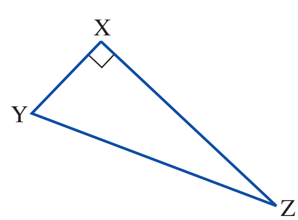
1 WE1 Name the hypotenuse in each of the following triangles.

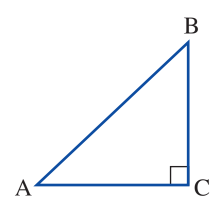
a



b



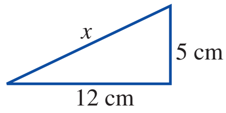
c



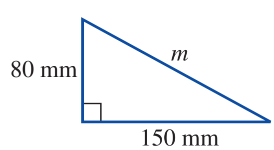
eBook*plus* **Digital doc** SkillSHEET 13.2 doc-1622 **Using Pythagoras' theorem**

2 WE2 Find the length of the hypotenuse in each of the following triangles.

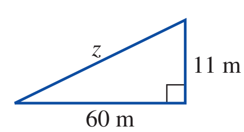
a



b



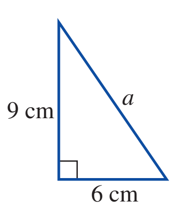
c



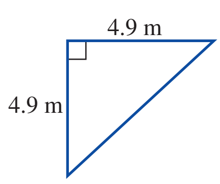
eBook*plus* **Digital doc** EXCEL Spreadsheet doc-1623 **Pythagoras**

3 In each of the following, find the length of the hypotenuse, correct to 2 decimal places.

a



b



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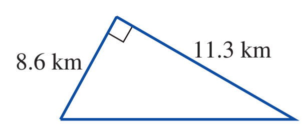
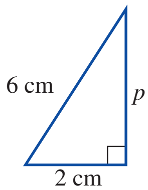


4 **WE3** Find the length of each shorter side in the right-angled triangles below, correct to 1 decimal place.

a



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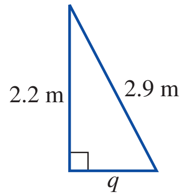
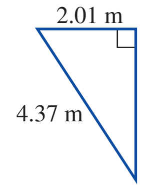


c



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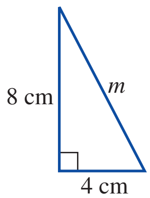
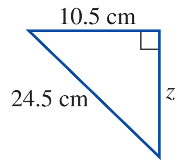
5 In each of the following right-angled triangles, find the length of the side marked with a pronumeral, correct to 1 decimal place.

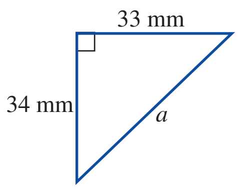
a



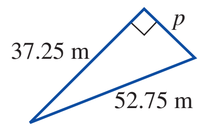
b



c

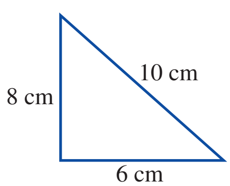


d

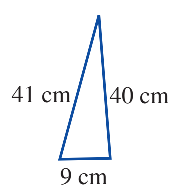


6 **WE4** Use the converse of Pythagoras' theorem to determine if the following triangles are right angled.

a



b



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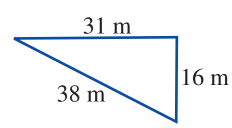
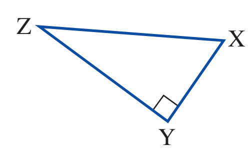


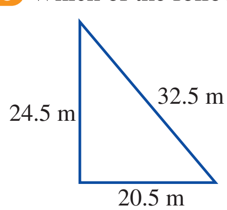
7 **MC** The hypotenuse in $\triangle XYZ$ at right is:

A XY **B** XZ
C YZ **D** impossible to tell

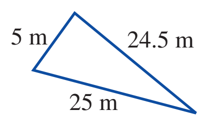


8 **MC** Which of the following triangles is right angled?

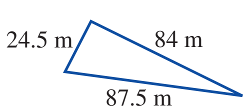
A



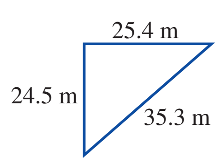
B



C



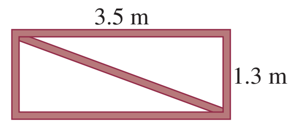
D



9 **WE5** A television antenna is 12 m high. To support it, wires are attached to the ground 5 m from the foot of the antenna. Find the length of each wire.

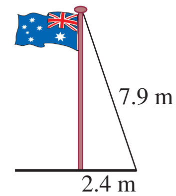
10 Susie needs to clean the guttering on her roof. She places her ladder 1.2 m back from the edge of the guttering that is 3 m above the ground. How long will Susie's ladder need to be (correct to 2 decimal places)?

11 A rectangular gate is 3.5 m long and 1.3 m wide. The gate is to be strengthened by a diagonal brace as shown at right. How long should the brace be (correct to 2 decimal places)?



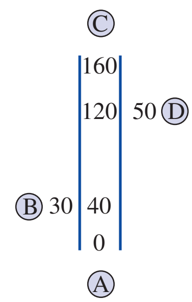
12 A 2.5 m ladder learns against a brick wall. The foot of the ladder is 1.2 m from the foot of the wall. How high up the wall will the ladder reach (correct to 1 decimal place)?

13 Use the measurements in the diagram at right to find the height of the flagpole, correct to 1 decimal place.



14 An isosceles, right-angled triangle has a hypotenuse of 10 cm. Calculate the length of the shorter sides. (*Hint:* Call both shorter sides x.)

15 A block of land was surveyed and the field diagram is shown.
a Draw a scale diagram of the block of land.
b Use Pythagoras' theorem to calculate the perimeter of the block of land, correct to the nearest metre.

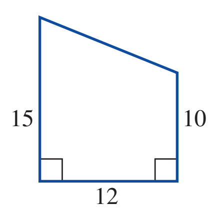


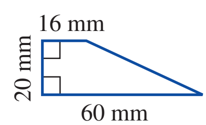
Further development

16 An aeroplane is flying at an altitude of 5000 m when it is 4 km from the end of the runway. What is the distance in metres from the aeroplane to the runway?

17 A gate is 2 m wide by 900 mm high. What is the length of a diagonal brace?

18 Each of the following figures has a sloping side whose length is unknown. Find the missing side length by first constructing a right-angled triangle.

a


b


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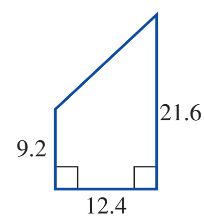
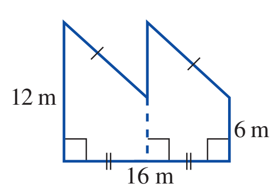
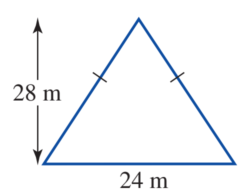
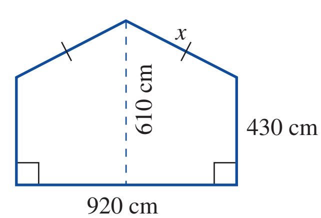
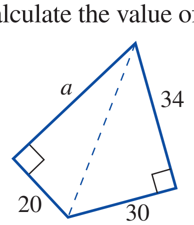


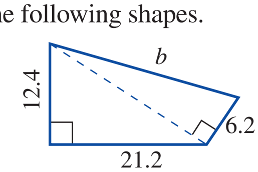
d


e


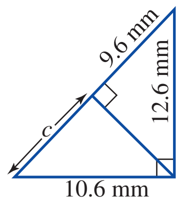
f


19 Calculate the value of the pronumerals in each of the following shapes.

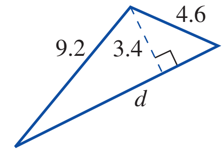
a


b


c

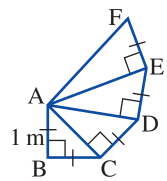


d



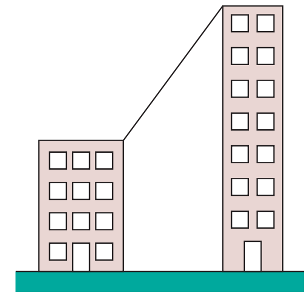
20 Consider the figure below. Find the length of:

a AC
b AD
c AE
d AF.



Leave each answer in square root form.

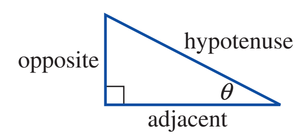
21 The heights of the buildings shown in the diagram at right are 40 metres and 80 metres respectively. The buildings are 40 metres apart. Find the length of a cable that would be needed to connect the roofs of the two buildings.



13B Calculating trigonometric ratios

In the previous section we looked at Pythagoras' theorem. This enabled us to find the length of one side of a right-angled triangle given the length of the other two. To use Pythagoras' theorem, we had to recognise the hypotenuse in a right-angled triangle.

In **trigonometry**, we need to be able to name the two shorter sides as well. We do this with reference to a given angle, and label them **opposite** and **adjacent**. They are the sides opposite and adjacent to the given angle. The diagram shows the sides labelled with respect to the angle, θ.



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Digital doc
EXCEL Spreadsheet
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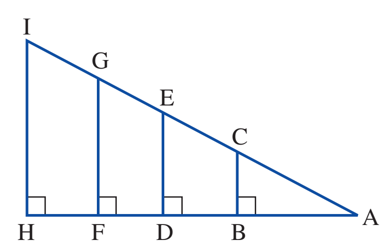
Tangent

INVESTIGATE: Looking at the tangent ratio

The tangent ratio is a ratio of sides in similar right-angled triangles, such as those in the diagram. ∠BAC is common to each triangle and is equal to 30°. We are going to look at the ratio of the opposite side to the adjacent side in each triangle. You can do this either on your calculator or by completing the spreadsheet 'Tangent' from the *Maths Quest General Mathematics Preliminary Course* ebook.



Complete each of the following measurements and calculations.

1 a BC = _______ mm **b** AB = _______ mm **c** $\frac{BC}{AB}$ = _______

2 a DE = _______ mm **b** AD = _______ mm **c** $\frac{DE}{AD}$ = _______

3 a FG = _______ mm **b** AF = _______ mm **c** $\frac{FG}{AF}$ = _______

4 a HI = _______ mm **b** AH = _______ mm **c** $\frac{HI}{AH}$ = _______

Remember that $\angle BAC$ is common to each triangle. In each of the above, part **c** is the ratio of the opposite side to the adjacent side of $\angle BAC$. What do you notice about each of these answers?

Trigonometry uses the ratio of side lengths to calculate the lengths of sides and the size of angles. The ratio of the opposite side to the adjacent side is called the **tangent ratio**. This ratio is fixed for any particular angle.

The tangent ratio for any angle, θ, can be found using the result:

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

In the investigation on page 421, we found that for a 30° angle the ratio was 0.58. We can find a more accurate value for the tangent ratio on a calculator by pressing tan and entering 30.

For all calculations in trigonometry you will need to make sure that your calculator is in **DEGREES MODE**. For most calculators you can check this by looking for a **DEG** in the display.

When measuring angles:

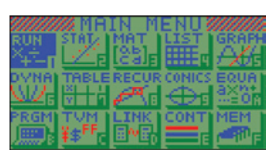
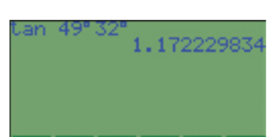
1 degree = 60 minutes
1 minute = 60 seconds

You need to be able to enter angles using both degrees and minutes into your calculator. Most calculators use a DMS (Degrees, Minutes, Seconds) button or a °'" button. Check with your teacher to see how to do this.

WORKED EXAMPLE 6

Using your calculator, find the following, correct to 3 decimal places.

a $\tan 60°$ **b** $15 \tan 75°$ **c** $\dfrac{8}{\tan 69°}$ **d** $\tan 49°32'$

THINK	WRITE/DISPLAY
a Press tan and enter 60.	**a** $\tan 60° = 1.732$
b Enter 15, press × and tan, enter 75.	**b** $15 \tan 75° = 55.981$
c Enter 8, press ÷ and tan, enter 69.	**c** $\dfrac{8}{\tan 69°} = 3.071$
Method 1	
d Press tan, enter 49, press DMS, enter 32, press DMS.	**d** $\tan 49°32' = 1.172$
Method 2	
1 From the **MENU** select **RUN**.	
2 To set the calculator up to accept degrees, minutes and even seconds, press OPTN, F6 for more options, then F5 for **ANGL**. The screen should appear as shown.	
3 Now to find the trigonometric ratio, enter **tan 49** and press F4 for degrees; enter **32** and press F4 for minutes; then press EXE.	

The tangent ratio is used to solve problems involving the opposite side and the adjacent side of a right-angled triangle. The tangent ratio does not allow us to solve problems that involve the hypotenuse.

The sine ratio (abbreviated to sin) is the name given to the ratio of the opposite side and the hypotenuse.

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EXCEL Spreadsheet
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Sine

INVESTIGATE: Looking at the sine ratio

The tangent ratio is the ratio of the opposite side and the adjacent side in a right-angled triangle. The sine ratio is the ratio of the opposite side and the hypotenuse. Look back to the right-angled triangles used in the tangent investigation on page 421.

Complete each of the following measurements and calculations by using your calculator or the spreadsheet 'Sine' from the *Maths Quest General Mathematics Preliminary Course* eBookPLUS.

As we saw earlier, ∠BAC is common to all of these similar triangles, and so in this exercise, we look at the ratio of the side opposite ∠BAC to the hypotenuse of each triangle.

1 **a** BC = _______ mm **b** AC = _______ mm **c** $\frac{BC}{AC}$ = _______

2 **a** DE = _______ mm **b** AE = _______ mm **c** $\frac{DE}{AE}$ = _______

3 **a** FG = _______ mm **b** AG = _______ mm **c** $\frac{FG}{AG}$ = _______

4 **a** HI = _______ mm **b** AI = _______ mm **c** $\frac{HI}{AI}$ = _______

In this exercise, part **c** is the ratio of the opposite side to ∠BAC to the hypotenuse. You should again notice that the answers are the same (or very close, allowing for measurement error).

In any right-angled triangle with equal angles, the ratio of the opposite side to the hypotenuse will remain the same, regardless of the size of the triangle. The formula for the **sine ratio** is:

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

The value of the sine ratio for any angle is found using the **sin** function on the calculator.

$$\sin 30° = 0.5$$

Check this on your calculator.

WORKED EXAMPLE 7

Find, correct to 3 decimal places:

a **sin 57°** **b** **9 sin 45°** **c** $\mathbf{\frac{18}{\sin 44°}}$ **d** **9.6 sin 26°12′.**

THINK	WRITE/DISPLAY
a Press [sin] and enter 57.	**a** $\sin 57° = 0.839$
b Enter 9, press [×] and [sin], enter 45.	**b** $9\sin 45° = 6.364$
c Enter 18, press [÷] and [sin], enter 44.	**c** $\frac{18}{\sin 44°} = 25.912$
d Enter 9.6, press [×] and [sin], enter 26, press [DMS], enter 12, press [DMS].	**d** $9.6\sin 26°12' = 4.238$

A third trigonometric ratio is the cosine ratio. This ratio compares the length of the adjacent side and the hypotenuse.

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EXCEL
Spreadsheet
doc-1628
Cosine

INVESTIGATE: Looking at the cosine ratio

Look back to the right-angled triangles used in the tangent investigation on page 421.

Complete each of the following measurements and calculations. You may do so by using the spreadsheet 'Cosine' from the *Maths Quest General Mathematics Preliminary Course* eBookPLUS.

1 a AB = _______ mm b AC = _______ mm c $\frac{AB}{AC}$ = _______

2 a AD = _______ mm b AE = _______ mm c $\frac{AD}{AE}$ = _______

3 a AF = _______ mm b AG = _______ mm c $\frac{AF}{AG}$ = _______

4 a AH = _______ mm b AI = _______ mm c $\frac{AH}{AI}$ = _______

Again for part **c**, you should get the same answer for each triangle. In each case, this is the cosine ratio of the common angle BAC.

The **cosine ratio** is found using the formula:

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

To calculate the cosine ratio for a given angle on your calculator, use the **cos** function. On your calculator check the calculation:

$$\cos 30° = 0.866$$

WORKED EXAMPLE 8

Find, correct to 3 decimal places:

a $\cos 27°$ **b** $6\cos 55°$ **c** $\dfrac{21.3}{\cos 74°}$ **d** $\dfrac{4.5}{\cos 82°46'}$.

THINK	WRITE/DISPLAY
a Press $\boxed{\cos}$ and enter 27.	**a** $\cos 27° = 0.891$
b Enter 6, press $\boxed{\times}$ and $\boxed{\cos}$, enter 55.	**b** $6\cos 55° = 3.441$
c Enter 21.3, press $\boxed{\div}$ and $\boxed{\cos}$, enter 74.	**c** $\dfrac{21.3}{\cos 74°} = 77.275$
d Enter 4.5, press $\boxed{\times}$ and $\boxed{\cos}$, enter 82, press $\boxed{\text{DMS}}$, enter 46, press $\boxed{\text{DMS}}$.	**d** $\dfrac{4.5}{\cos 82°46'} = 35.740$

Similarly, if we are given the sin, cos or tan of an angle, we are able to calculate the size of that angle using the calculator. We do this using the inverse functions. On most calculators these are the 2nd function of the sin, cos and tan functions and are denoted $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$.

WORKED EXAMPLE 9

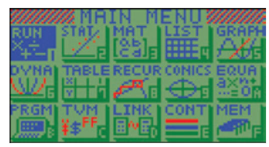
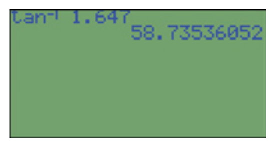
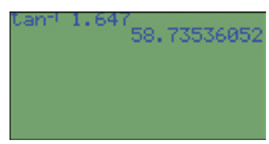
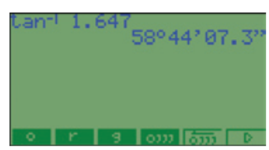
Find θ, correct to the nearest degree, given that $\sin\theta = 0.738$.

THINK	WRITE/DISPLAY
1 Press $\boxed{\text{2nd F}}$ [$\sin^{-1}$] and enter 0.738.	
2 Round your answer to the nearest degree.	$\theta = 48°$

So far, we have dealt only with angles that are whole degrees. You need to be able to make calculations using minutes as well. On most calculators, you will use the **DMS** (Degrees, Minutes, Seconds) function or the °’’’ function.

WORKED EXAMPLE 10

Given that $\tan\theta = 1.647$, calculate θ to the nearest minute.

THINK	WRITE/DISPLAY
Method 1	
1 Press 2nd F [**tan^{-1}**] and enter 1.647.	
2 Convert your answer to degrees and minutes by pressing DMS.	$\theta = 58°44'$
Method 2	
1 From the **MENU** select **RUN**.	
2 As with a scientific calculator, press SHIFT [**tan^{-1}**] and enter 1.647, then press EXE.	
3 Display the angle options by pressing OPTN, F6 for more choices, and then F5 for **ANGL**.	
4 The function for getting the answer displayed in degrees, minutes and seconds is accessed by pressing F5.	

REMEMBER

1. The tangent ratio is the ratio of the opposite side and the adjacent side.
$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$
2. The sine ratio is the ratio of the opposite side and the hypotenuse.
$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$
3. The cosine ratio is the ratio of the adjacent side and the hypotenuse.
$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$
4. The value of the trigonometric ratios can be found using the **sin**, **cos** and **tan** functions on your calculator.
5. The angle can be found when given the trigonometric ratio using the **sin^{-1}**, **cos^{-1}** and **tan^{-1}** functions on your calculator.

EXERCISE

13B Calculating trigonometric ratios

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Rounding to a given number of decimal places

1 WE6 Calculate the value of each of the following, correct to 3 decimal places.

a $\tan 57°$ **b** $9 \tan 63°$ **c** $\dfrac{8.6}{\tan 12°}$ **d** $\tan 33°19'$

2 WE7 Calculate the value of each of the following, correct to 3 decimal places.

a $\sin 37°$ **b** $9.3 \sin 13°$ **c** $\dfrac{14.5}{\sin 72°}$ **d** $\dfrac{48}{\sin 67°40'}$

3 WE8 Calculate the value of each of the following, correct to 3 decimal places.

a $\cos 45°$ **b** $0.25 \cos 9°$ **c** $\dfrac{6}{\cos 24°}$ **d** $5.9 \cos 2°3'$

4 Calculate the value of each of the following, correct to 4 significant figures.

a $\sin 30°$ **b** $\cos 15°$ **c** $\tan 45°$

d $48 \tan 85°$ **e** $128 \cos 60°$ **f** $9.35 \sin 8°$

g $\dfrac{4.5}{\cos 32°}$ **h** $\dfrac{0.5}{\tan 20°}$ **i** $\dfrac{15}{\sin 72°}$

5 Calculate the value of each of the following, correct to 2 decimal places.

a $\sin 24°38'$ **b** $\tan 57°21'$ **c** $\cos 84°40'$

d $9 \cos 55°30'$ **e** $4.9 \sin 35°50'$ **f** $2.39 \tan 8°59'$

g $\dfrac{19}{\tan 67°45'}$ **h** $\dfrac{49.6}{\cos 47°25'}$ **i** $\dfrac{0.84}{\sin 75°5'}$

6 WE9 Find θ, correct to the nearest degree, given that $\sin \theta = 0.167$.

7 Find θ, correct to the nearest degree, given that:
a $\sin \theta = 0.698$ **b** $\cos \theta = 0.173$ **c** $\tan \theta = 1.517$.

8 WE10 Find θ, correct to the nearest minute, given that $\cos \theta = 0.058$.

9 Find θ, correct to the nearest minute, given that:
a $\tan \theta = 0.931$ **b** $\cos \theta = 0.854$ **c** $\sin \theta = 0.277$.

Further development

10 Find the value of each of the following trigonometric ratio pairs. Give your answers correct to four decimal places.

a $\sin 40°, \cos 50°$ **b** $\sin 70°, \cos 20°$ **c** $\sin 13°, \cos 77°$ **d** $\sin 84°, \cos 6°$

11 What did you notice about the relationship between sin and cos in question **10**? Use this to complete each of the following.

a $\sin 30° = \cos$ ___ **b** $\cos 75° = \sin$ ___ **c** $\sin 28° =$ ____ **d** $\cos 45° = \sin$ ___

12 Find:

a $\sin 23°$ **b** $\cos 23°$ **c** $\dfrac{\sin 23°}{\cos 23°}$ **d** $\tan 23°$.

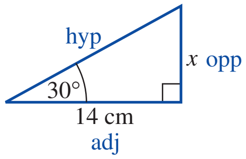
13 Find:

a $\sin 67°$ **b** $\cos 67°$ **c** $(\sin 67°)^2 + (\cos 67°)^2$.

14 Use your answer to question **13** to find $(\sin 34°)^2 + (\cos 34°)^2$. Check your answer with your calculator.

15 Fred tries to solve $\sin \theta = 1.2$ on his calculator, however an error statement is returned.
a Explain why there is no solution to this question.
b What is the only trigonometric ratio that can possibly equal 1.2?

13C Finding an unknown side



We can use the trigonometric ratios to find the length of one side of a right-angled triangle if we know the length of another side and an angle. Consider the triangle at right.

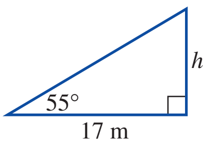
In this triangle we are asked to find the length of the opposite side and have been given the length of the adjacent side.

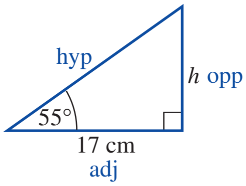
We know from the formula that: $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$. In this example, $\tan 30° = \frac{x}{14}$. From our calculator we know that $\tan 30° = 0.577$. We can set up an equation that will allow us to find the value of x.

$$\begin{aligned}\tan\theta &= \frac{\text{opp}}{\text{adj}}\\ \tan 30° &= \frac{x}{14}\\ x &= 14\tan 30°\\ &= 8.083\text{ cm}\end{aligned}$$

WORKED EXAMPLE 11

Use the tangent ratio to find the value of h in the triangle at right, correct to 2 decimal places.



THINK	WRITE
1 Label the sides of the triangle opp, adj and hyp.	
2 Write the tangent formula.	$\tan\theta = \frac{\text{opp}}{\text{adj}}$
3 Substitute for θ (55°) and the adjacent side (17 m).	$\tan 55° = \frac{h}{17}$
4 Make h the subject of the equation.	$h = 17\tan 55°$
5 Calculate.	$= 24.28$ cm

In the example above, we were told to use the tangent ratio. In practice, we need to be able to look at a problem and then decide if the solution is found using the sin, cos or tan ratio. To do this we need to examine the three formulas.

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

We use the tan ratio when we are finding either the length of the opposite or adjacent side and are given the length of the other.

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

The sin ratio is used when we are finding the length of the opposite side or the hypotenuse and are given the length of the other.

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

The cos ratio is for problems where we are finding the length of the adjacent side or the hypotenuse and are given the length of the other.

To make the decision we need to label the sides of the triangle and make a decision based on these labels.

WORKED EXAMPLE 12

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Tutorial
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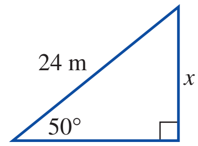
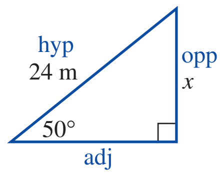
Worked example 12

Find the length of the side marked x, correct to 2 decimal places.



THINK | **WRITE**

1 Label the sides of the triangle.



2 x is the opposite side and 24 m is the hypotenuse, therefore use the sin formula.

$$\sin\theta = \frac{\text{opp}}{\text{adj}}$$

3 Substitute for θ and the hypotenuse.

$$\sin 50° = \frac{x}{24}$$

Method 1

4 Make x the subject of the equation.

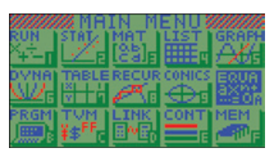
$$x = 24\sin 50°$$

5 Calculate.

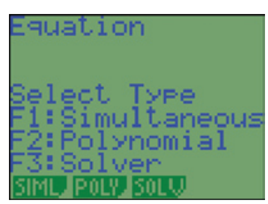
$$= 18.39 \text{ m}$$

Method 2

1 From the **MENU** select **EQUA**.

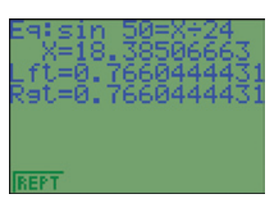


2 Press F3 for **Solver**.



3 Delete any existing equation and enter $\sin 50° = \frac{x}{24}$ by pressing **sin 50** SHIFT = **X** ÷ **24** EXE.
Do not worry about a different value of X in the display at this stage as it is a previously stored value.

4 Press F6 for **SOLV** to solve this equation.



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Interactivity
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SOHCAHTOA

To remember each of the formulas more easily, we can use this acronym:

SOHCAHTOA

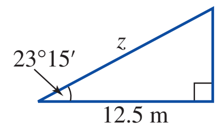
We pronounce this acronym as 'Sock ca toe her'. The initials of the acronym represent the three trigonometric formulas.

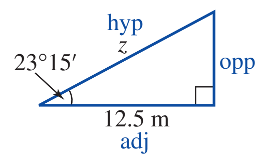
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

Care needs to be taken at the substitution stage. In the above examples, the unknown side was the numerator in the fraction, hence we multiplied to find the answer. If after substitution, the unknown side is in the denominator, the final step is done by division.

WORKED EXAMPLE 13

Find the length of the side marked z in the triangle at right.



THINK	WRITE
1 Label the sides opp, adj and hyp.	
2 Choose the cosine ratio because we are finding the hypotenuse and have been given the adjacent side.	
3 Write the formula.	$\cos\theta = \frac{\text{adj}}{\text{hyp}}$
4 Substitute for θ and the adjacent side.	$\cos 23°15' = \frac{12.5}{z}$
5 Make z the subject of the equation.	$z\cos 23°15' = 12.5$ $z = \frac{12.5}{\cos 23°15'}$
6 Calculate.	$= 13.60$ m

If you are using the graphics calculator to solve an equation that involves the use of degrees and minutes, the angle needs to be entered as a fraction. In worked example 13 above, cos 23°15′ needs to be entered as $\cos 23\frac{15}{60}$ as the equation solver does not allow access to the degrees, minutes and seconds function.

Trigonometry is used to solve many practical problems. In these cases, it is necessary to draw a diagram to represent the problem and then use trigonometry to solve the problem. With written problems that require you to draw the diagram, it is necessary to give the answer in words.

WORKED EXAMPLE 14

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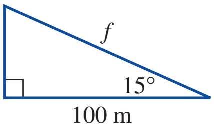
Tutorial
int-2338
Worked example 14

A flying fox is used in an army training camp. The flying fox is supported by a cable that runs from the top of a cliff face to a point 100 m from the base of the cliff. The cable makes a 15° angle with the horizontal. Find the length of the cable used to support the flying fox.

THINK | **WRITE**

1 Draw a diagram and show information.



2 Label the sides of the triangle opp, adj and hyp.

3 Choose the cosine ratio because we are finding the hypotenuse and have been given the adjacent side.

4 Write the formula.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

5 Substitute for θ and the adjacent side.

$$\cos 15° = \frac{100}{f}$$

6 Make f the subject of the equation.

$$f\cos 15° = 100$$

$$f = \frac{100}{\cos 15°}$$

7 Calculate.

$$= 103.5 \text{ m}$$

8 Give a written answer.

The cable is approximately 103.5 m long.

REMEMBER

1. Trigonometry can be used to find the length of a side in a right-angled triangle when we are given the length of one side and the size of an angle.
2. The trigonometric formulas are:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

3. Take care to choose the correct trigonometric ratio for each question.
4. Substitute carefully and note the change in the calculation, depending upon whether the unknown side is in the numerator or denominator.
5. Before using your calculator, check that it is in degrees mode.
6. Be sure that you know how to enter degrees and minutes into your calculator.
7. Worded problems will require you to draw a diagram and give a written answer.

EXERCISE **13C**

Finding an unknown side

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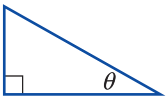
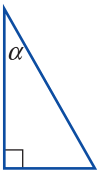
SkillSHEET 13.4
doc-1630
Solving equations of the type $a = \frac{x}{b}$ to find x

1 Label the sides of each of the following triangles, with respect to the angle marked with the pronumeral.

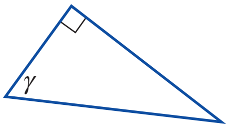
a



b



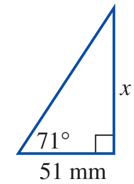
c



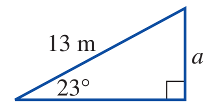
eBook*plus*

Digital doc
SkillSHEET 13.5
doc-1631
Solving equations of the type $a = \frac{b}{x}$ to find x

2 WE11 Use the tangent ratio to find the length of the side marked x (correct to 1 decimal place).



3 Use the sine ratio to find the length of the side marked a (correct to 2 decimal places).



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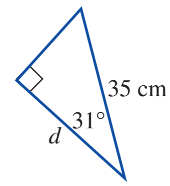
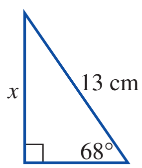
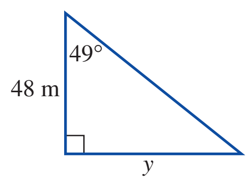
SkillSHEET 13.6
doc-1632
Rearranging formulas

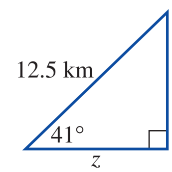
4 Use the cosine ratio to find the length of the side marked d (correct to 3 significant figures).



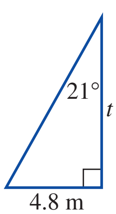
5 WE12 The following questions use the tan, sin or cos ratios in their solution. Find the size of the side marked with the pronumeral, correct to 3 significant figures.

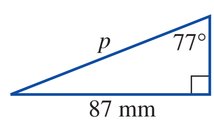
a


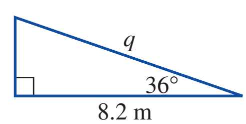
b


c


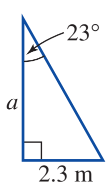
6 WE13 Find the length of the side marked with the pronumeral in each of the following (correct to 1 decimal place).

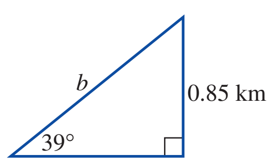
a


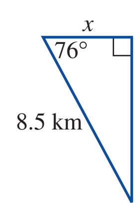
b


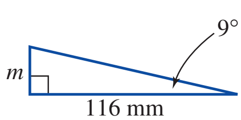
c


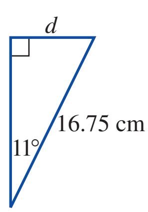
7 Find the length of the side marked with the pronumeral in each of the following (correct to 3 significant figures).

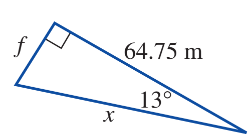
a


b


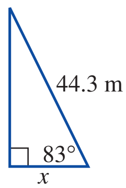
c


d


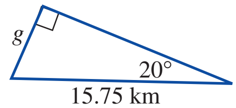
e


f


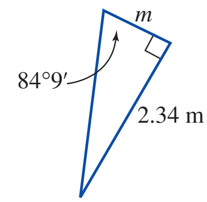
g



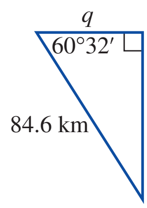
h



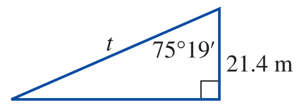
i



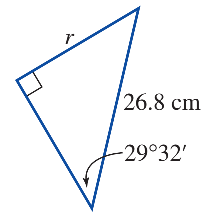
j



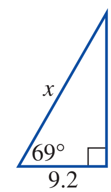
k



l



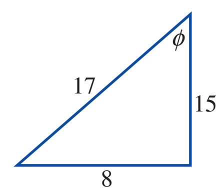
8 **MC** Look at the diagram at right and state which of the following is correct.



A $x = 9.2 \sin 69°$ **B** $x = \dfrac{9.2}{\sin 69°}$

C $x = 9.2 \cos 69°$ **D** $x = \dfrac{9.2}{\cos 69°}$

9 **MC** Study the triangle at right and state which of the following is correct.



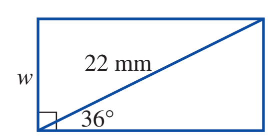
A $\tan \phi = \frac{8}{15}$ **B** $\tan \phi = \frac{15}{8}$

C $\sin \phi = \frac{15}{17}$ **D** $\cos \phi = \frac{8}{17}$

10 **MC** Which of the statements below is *not* correct?

A The value of tan θ can never be greater than 1.
B The value of sin θ can never be greater than 1.
C The value of cos θ can never be greater than 1.
D $\tan 45° = 1$

11 **MC** Study the diagram at right and state which of the statements is correct.



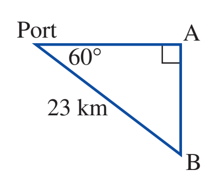
A $w = 22 \cos 36°$ **B** $w = \dfrac{22}{\sin 36°}$

C $w = 22 \cos 54°$ **D** $w = 22 \sin 54°$

12 **WE14** A tree casts a 3.6 m shadow when the sun's angle of elevation is 59°. Calculate the height of the tree, correct to the nearest metre.

13 A 10 m ladder just reaches to the top of a wall when it is leaning at 65° to the ground. How far from the foot of the wall is the ladder (correct to 1 decimal place)?

14 The diagram at right shows the paths of two ships, A and B, after they have left port. If ship B sends a distress signal, how far must ship A sail to give assistance (to the nearest kilometre)?



15 A rectangle 13.5 cm wide has a diagonal that makes a 24° angle with the horizontal.

a Draw a diagram of this situation.
b Calculate the width of the rectangle, correct to 1 decimal place.

16 A wooden gate has a diagonal brace built in for support. The gate stands 1.4 m high and the diagonal makes a 60° angle with the horizontal.

a Draw a diagram of the gate.

b Calculate the length that the diagonal brace needs to be.

17 The wire support for a flagpole makes a 70° angle with the ground. If the support is 3.3 m from the base of the flagpole, calculate the length of the wire support (correct to 2 decimal places).

18 A ship drops anchor vertically with an anchor line 60 m long. After one hour the anchor line makes a 15° angle with the vertical.

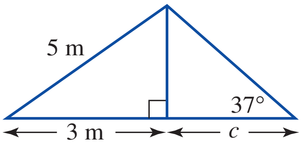
a Draw a diagram of this situation.

b Calculate the depth of water, correct to the nearest metre.

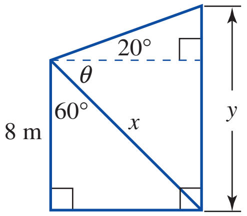
c Calculate the distance that the ship has drifted, correct to 1 decimal place.

Further development

19 Find the length of the side marked 'c' in the triangle at right.

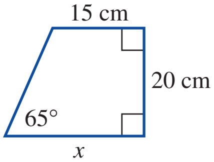


20 In the diagram at right find the size of angle θ (to the nearest degree) and the side lengths 'x' and 'y' correct to one decimal place.

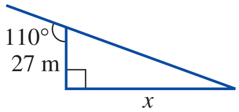


21 Find the value of the unknown sides in each of the following:

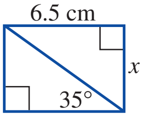
a



b



c



22 A boat is moored in calm water with a depth of 29 m. If the anchor line makes a 28° angle with the surface of the water, what is the length of the anchor line?

23 A person hopes to swim across a river of width 43 metres. When she swims across the current pushes her downstream to a point such that the swim line makes a 67° angle with the river bank. Calculate:

a how far she swam

b how far she finished from the planned finishing point.

24 Peter says that whenever finding the hypotenuse of a right-angled triangle, the solution will involve dividing by the trigonometric ratio. Is Peter correct? Explain your answer.

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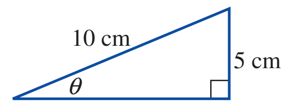
13D Finding angles

In this chapter so far, we have concerned ourselves with finding side lengths. We are also able to use trigonometry to find the sizes of angles when we have been given side lengths. We need to reverse our previous processes.

Consider the triangle at right. We want to find the size of the angle marked θ.



Using the formula $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ we know that in this triangle

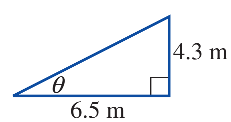
$$\begin{aligned}\sin\theta &= \tfrac{5}{10}\\ &= \tfrac{1}{2}\\ &= 0.5\end{aligned}$$

We then calculate $\sin^{-1}(0.5)$ to find that $\theta = 30°$.

As with all trigonometry it is important that you have your calculator set to degrees mode.

WORKED EXAMPLE 15

Find the size of angle θ, correct to the nearest degree, in the triangle at right.



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Tutorial
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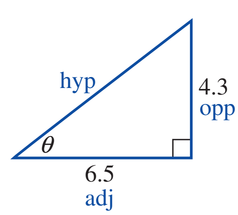
Worked example 15

THINK | **WRITE**

Method 1

1 Label the sides of the triangle and choose the tan ratio.



$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

2 Substitute for the opposite and adjacent sides in the triangle and simplify.

$= \dfrac{4.3}{6.5}$

$= 0.6615$

3 Make θ the subject of the equation.

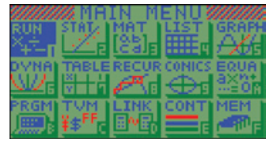
$\theta = \tan^{-1}(0.6615)$

4 Calculate.

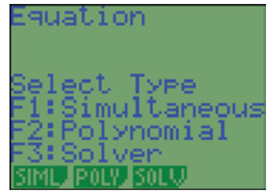
$= 33°$ (to the nearest degree)

Method 2

1. From the **MENU** select **EQUA**.



2. Press F3 for **Solver**.



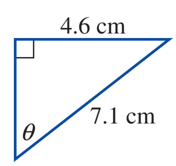
3. Delete any existing equation and enter $\tan x = \frac{4.3}{6.5}$ by pressing tan X SHIFT [=] **4.3** ÷ **6.5** EXE.

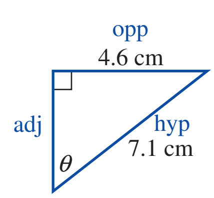
4. Press F6 for **SOLV** to solve this equation.

In many cases we will need to calculate the size of an angle, correct to the nearest minute. The same method for finding the solution is used; however, you will need to use your calculator to convert to degrees and minutes.

WORKED EXAMPLE 16

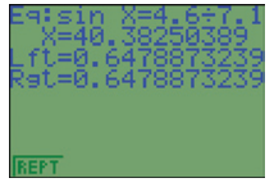
Find the size of the angle θ, correct to the nearest minute.



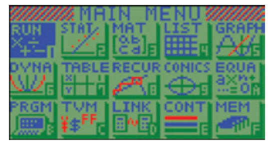
THINK	WRITE
Method 1	
1. Label the sides of the triangle and choose the sin ratio.	 $\sin\theta = \frac{\text{opp}}{\text{hyp}}$
2. Substitute for the opposite side and adjacent in the triangle and simplify.	$= \frac{4.6}{7.1}$ $= 0.6479$
3. Make θ the subject of the equation.	$\theta = \sin^{-1}(0.6479)$
4. Calculate and convert your answer to degrees and minutes.	$= 40°23'$ (to the nearest minute)

Method 2

1. Solve the equation as shown previously, which gives an answer in degrees as a decimal.

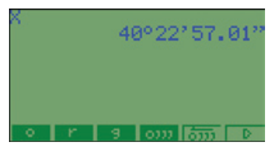


2. Press MENU and select **RUN**.



3. Press X EXE to recall the value of X from the equation.

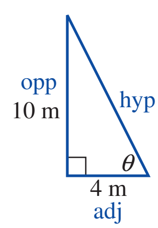
4. To access the angle functions, press OPTN F6, F5 for **ANGL**, and F5 again to convert to degrees, minutes and seconds.



The same methods can be used to solve problems. As with finding sides, we set the question up by drawing a diagram of the situation.

WORKED EXAMPLE 17

A ladder is leant against a wall. The foot of the ladder is 4 m from the base of the wall and the ladder reaches 10 m up the wall. Calculate the angle that the ladder makes with the ground.

THINK	WRITE
1 Draw a diagram and label the sides.	
2 Choose the tangent ratio and write the formula.	$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$
3 Substitute for the opposite and adjacent side, then simplify.	$= \dfrac{10}{4}$ $= 2.5$
4 Make θ the subject of the equation.	$\theta = \tan^{-1}(2.5)$
5 Calculate.	$= 68°12'$
6 Give a written answer.	The ladder makes an angle of 68°12′ with the ground.

REMEMBER

1. Make sure that the calculator is in degrees mode.
2. To find an angle given the trigonometric ratio, press SHIFT and then the appropriate ratio button.
3. Be sure to know how to get your calculator to display an answer in degrees and minutes. When rounding off minutes, check if the number of seconds is greater than 30.
4. When solving triangles remember the SOHCAHTOA rule to choose the correct formula.
5. In worded problems draw a diagram and give an answer written in words.

EXERCISE **13D**

Finding angles

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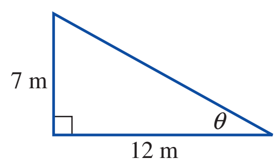
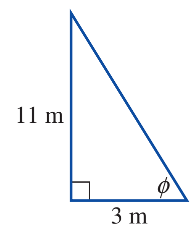
Digital doc SkillSHEET 13.7 doc-1634 **Rounding angles to the nearest degree**

1 In each of the following, use the tangent ratio to find the size of the angle marked with the pronumeral, correct to the nearest degree.

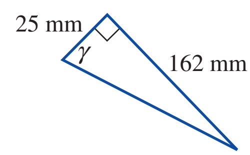
a



b



c



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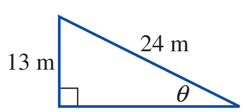
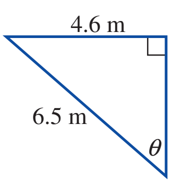
Digital doc SkillSHEET 13.8 doc-1635 **Rounding angles to the nearest minute**

2 In each of the following, use the sine ratio to find the size of the angle marked with the pronumeral, correct to the nearest minute.

a



b



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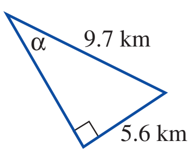

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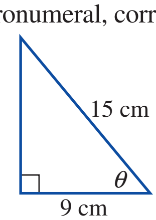
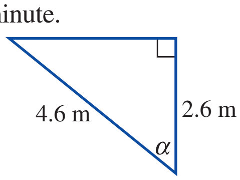
Digital doc SkillSHEET 13.9 doc-1636 **Rounding angles to the nearest second**

3 In each of the following, use the cosine ratio to find the size of the angle marked with the pronumeral, correct to the nearest minute.

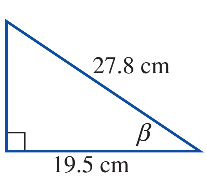
a



b

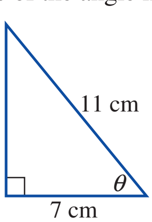


c



4 WE15 In the following triangles, you will need to use all three trigonometric ratios. Find the size of the angle marked θ, correct to the nearest degree.

a



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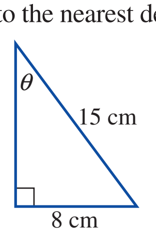

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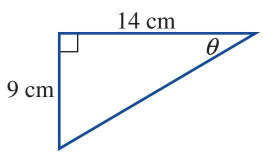

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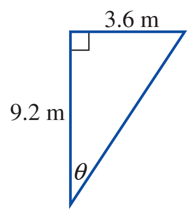
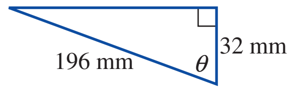


e


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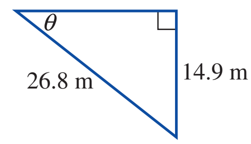
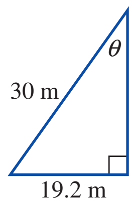


5 WE16 In each of the following, find the size of the angle marked θ, correct to the nearest minute.

a


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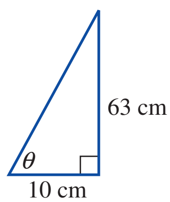

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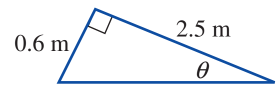
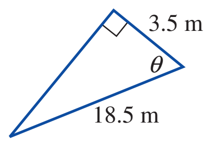


d


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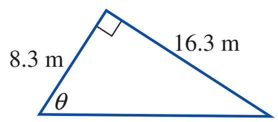

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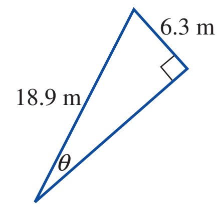
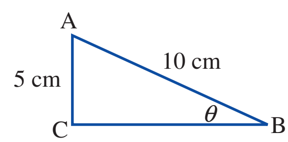


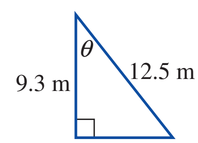
6 MC Look at the triangle drawn at right.
Which of the statements below is correct?

A $\angle ABC = 30°$ **B** $\angle ABC = 60°$
C $\angle CAB = 30°$ **D** $\angle ABC = 45°$



7 MC Consider the triangle drawn at right. θ is closest to:

A 41°55′ **B** 41°56′ **C** 48°4′ **D** 48°5′

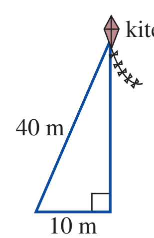


8 MC The exact value of $\sin \theta = \frac{\sqrt{3}}{2}$. The angle $\theta =$

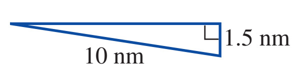
A 30° **B** 45° **C** 60° **D** 90°

9 WE17 A 10 m ladder leans against a 6 m high wall. Find the angle that the ladder makes with the horizontal, correct to the nearest degree.

10 A kite is flying on a 40 m string. The kite is flying 10 m away from the vertical as shown in the figure at right. Find the angle the string makes with the horizontal, correct to the nearest minute.

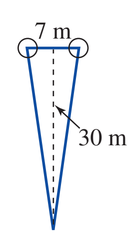


11 A ship's compass shows a course due east of the port from which it sails. After sailing 10 nautical miles, it is found that the ship is 1.5 nautical miles off course as shown in the figure below.



Find the error in the compass reading, correct to the nearest minute.

12 The diagram at right shows a footballer's shot at goal.

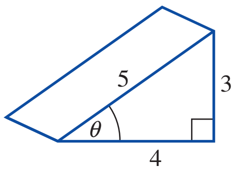


By dividing the isosceles triangle in half, calculate, to the nearest degree, the angle within which the footballer must kick to get the ball to go between the posts.

13 A golfer hits the ball 250 m, but 20 m off centre. Calculate the angle at which the ball deviated from a straight line, correct to the nearest minute.

Further development

14 MC The figure below shows a BMX bicycle ramp. All measurements are shown in metres.



The correct expression for the angle of elevation, θ, of the ramp is:

A $\sin^{-1}\left(\frac{4}{5}\right)$ B $\cos^{-1}\left(\frac{4}{5}\right)$

C $\tan^{-1}\left(\frac{4}{5}\right)$ D $\cos^{-1}\left(\frac{3}{5}\right)$

15 MC A flagpole that is 2 metres tall casts a shadow that is 0.6 metres long. The angle of the sun to the ground is:

A 70° B 71°
C 72° D 73°

16 A javelin that is 1.95 m long is thrown and sticks 20 cm into the ground. Given that the sun is directly overhead and that the javelin casts a 90 cm shadow, find the angle that the javelin makes with the ground.

17 A hot air balloon is hovering in strong winds 10 vertical metres above the ground. The balloon is being held in place by a rope that is 15 m long. What angle does the rope make with the ground?

18 A cable car follows a direct line from a mountain peak (altitude 1250 m) to a ridge (altitude 840 m). If the horizontal distance between the peak and the ridge is 430 m, calculate the angle through which the cable car descends.

19 A ramp joins two points 1.2 metres apart. One point is 25 cm higher than the other.
a Find the length of the ramp.
b Find the angle of inclination of the ramp.

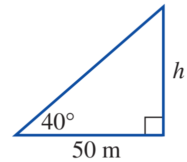
13E Angles of elevation and depression

The **angle of elevation** is measured upwards from a horizontal and refers to the angle at which we need to look up to see an object. Similarly, the **angle of depression** is the angle at which we need to look down from the horizontal to see an object.

We are able to use the angles of elevation and depression to calculate the heights and distances of objects that would otherwise be difficult to measure.

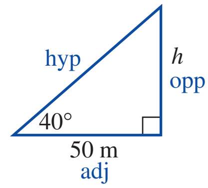
WORKED EXAMPLE 18

From a point 50 m from the foot of a building, the angle of elevation to the top of the building is measured as 40°.
Calculate the height, *h*, of the building, correct to the nearest metre.



THINK | **WRITE**

1. Label the sides of the triangle opp, adj and hyp.

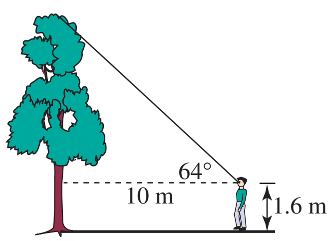


2. Choose the tangent ratio because we are finding the length of the opposite side and have been given the length of the adjacent side.
3. Write the formula. $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$
4. Substitute for θ and the adjacent side. $\tan 40° = \dfrac{h}{50}$
5. Make h the subject of the equation. $h = 50 \tan 40°$
6. Calculate. $= 42$ m
7. Give a written answer. The height of the building is approximately 42 m.

In practical situations, the angle of elevation is measured using a clinometer. Therefore, the angle of elevation is measured from a person's height at eye level. For this reason, the height at eye level must be added to the calculated answer.

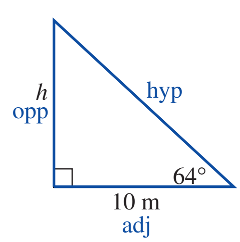
WORKED EXAMPLE 19

Bryan measures the angle of elevation to the top of a tree as 64°, from a point 10 m from the foot of the tree. If the height of Bryan's eyes is 1.6 m, calculate the height of the tree, correct to 1 decimal place.



THINK | **WRITE**

1. Label the sides opp, adj and hyp.



2. Choose the tangent ratio because we are finding the length of the opposite side and have been given the length of the adjacent side.
3. Write the formula. $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$
4. Substitute for θ and the adjacent side. $\tan 64° = \dfrac{h}{10}$
5. Make h the subject of the equation. $h = 10 \tan 64°$
6. Calculate h. $= 20.5$ m
7. Add the eye height. $20.5 + 1.6 = 22.1$
8. Give a written answer. The height of the building is approximately 22.1 m.

A similar method for finding the solution is used for problems that involve an angle of depression.

WORKED EXAMPLE 20

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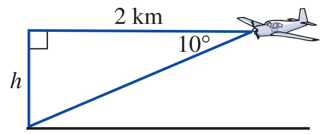
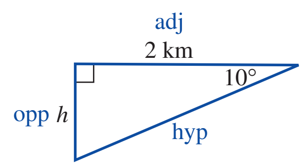
Tutorial
int-2340
Worked example 20

When an aeroplane is 2 km from a runway, the angle of depression to the runway is 10°. Calculate the altitude of the aeroplane, correct to the nearest metre.



THINK | **WRITE**

1. Label the sides of the triangle opp, adj and hyp.
2. Choose the tan ratio, because we are finding the length of the opposite side given the length of the adjacent side.



3. Write the formula.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

4. Substitute for θ and the adjacent side, converting 2 km to metres.

$$\tan 10° = \frac{h}{2000}$$

5. Make h the subject of the equation.

$$h = 2000 \tan 10°$$

6. Calculate.

$$= 353 \text{ m}$$

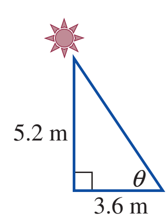
7. Give a written answer.

The altitude of the aeroplane is approximately 353 m.

Angles of elevation and depression can also be calculated by using known measurements. This is done by drawing a right-angled triangle to represent a situation.

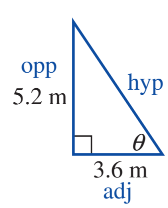
WORKED EXAMPLE 21

A 5.2 m building casts a 3.6 m shadow. Calculate the angle of elevation of the sun, correct to the nearest degree.



THINK | **WRITE**

1. Label the sides opp, adj and hyp.
2. Choose the tan ratio because we are given the length of the opposite and adjacent sides.



3 Write the formula. $\tan\theta = \frac{\text{opp}}{\text{adj}}$

4 Substitute for opposite and adjacent. $\tan\theta = \frac{5.2}{3.6}$

5 Make θ the subject of the equation. $\theta = \tan^{-1}\frac{5.2}{3.6}$

6 Calculate. $= 55°$

7 Give a written answer. The angle of elevation of the sun is approximately 55°.

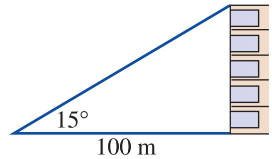
REMEMBER

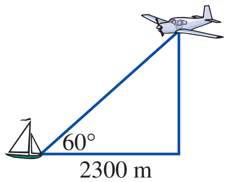
1. The angle of elevation is the angle at which you look up to see an object.
2. The angle of depression is the angle at which you look down to see an object.
3. Problems can be solved by using angles of elevation and depression with the aid of a diagram.
4. Worded problems should be given an answer written in words.

To capture the top of the building in this photo the photographer had to tilt the camera upwards, hence, increase the angle of elevation.

EXERCISE
13E Angles of elevation and depression

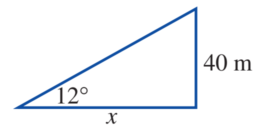
1 WE18 From a point 100 m from the foot of a building, the angle of elevation to the top of the building is 15°. Calculate the height of the building, correct to 1 decimal place.



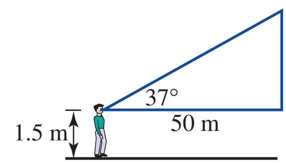


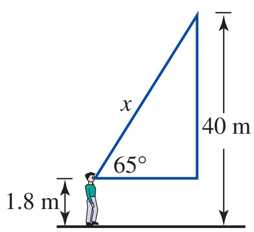
2 The angle of elevation from a ship to an aeroplane is 60°. The aeroplane is 2300 m due north of the ship. Calculate the altitude of the aeroplane, correct to the nearest metre.

3 From a point out to sea, a ship sights the top of a lighthouse at an angle of elevation of 12°. It is known that the top of the lighthouse is 40 m above sea level. Calculate the distance of the ship from the lighthouse, correct to the nearest 10 m.



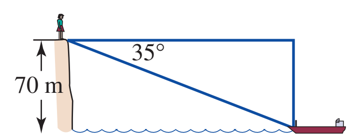
4 WE19 From a point 50 m from the foot of a building, Rod sights the top of a building at an angle of elevation of 37°. Given that Rod's eyes are at a height of 1.5 m, calculate the height of the building, correct to 1 decimal place.



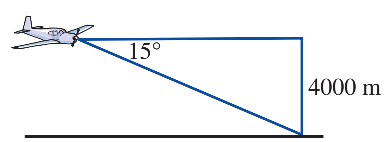


5 Richard is flying a kite and sights the kite at an angle of elevation of 65°. The altitude of the kite is 40 m and Richard's eyes are at a height of 1.8 m. Calculate the length of string the kite is flying on, correct to 1 decimal place.

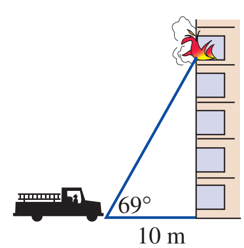
6 WE20 Bettina is standing on top of a cliff, 70 m above sea level. She looks directly out to sea and sights a ship at an angle of depression of 35°. Calculate the distance of the ship from shore, to the nearest metre.



7 From an aeroplane flying at an altitude of 4000 m, the runway is sighted at an angle of depression of 15°. Calculate the distance of the aeroplane from the runway, correct to the nearest kilometre.



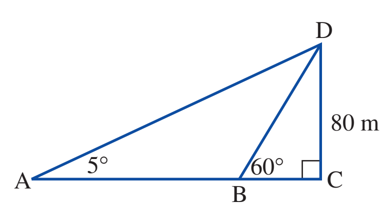
8 There is a fire on the fifth floor of a building. The closest a fire truck can get to the building is 10 m. The angle of elevation from this point to where people need to be rescued is 69°. If the fire truck has a 30 m ladder, can the ladder be used to make the rescue?



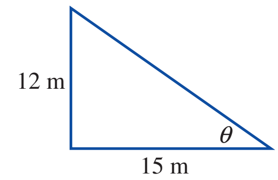
9 From a navy vessel, a beacon which is 80 m above sea level is sighted at an angle of elevation of 5°. The vessel sailed towards the beacon and thirty minutes later the beacon is at an angle of elevation of 60°.
Use the diagram on the right to complete the following.

a Calculate the distance that the vessel was from the beacon when the angle of elevation to the beacon was 5° (the distance AC).

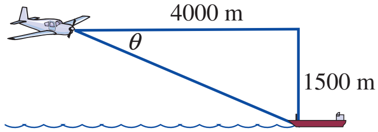
b Calculate the distance that the vessel sailed in the 30 minutes between the two readings.



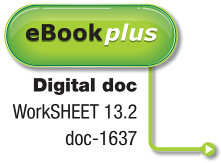
10 WE21 A 12 m high building casts a shadow 15 m long. Calculate the angle of elevation of the sun, to the nearest degree.



11 An aeroplane that is at an altitude of 1500 m is 4000 m from a ship in a horizontal direction, as shown below. Calculate the angle of depression from the aeroplane to the ship, to the nearest degree.



12 The angle of elevation to the top of a tower is 12° from a point 400 m from the foot of the tower.
  a Draw a diagram of this situation.
  b Calculate the height of the tower, correct to 1 decimal place.
  c Calculate the angle of elevation to the top of the tower from a point 100 m from the foot of the tower.



Further development

13 A lifesaver sits in a tower 2 m above sea level. He sees a swimmer having difficulty at an angle of depression of 12°. How far is the swimmer from the tower?

14 From the top of a lookout 50 m above the ground, the angle of depression to a campsite is 37°. How far is the camp from the base of the lookout?

15 A helicopter hovers 1800 metres above the ground. The angle of depression to two lost bushwalkers is 40° and 60° respectively. Calculate the distance between the two bushwalkers.

16 A ship sights a lighthouse at an angle of elevation of 50°. The lighthouse is 50 m above sea level.
  a Find the distance of the ship from the lighthouse.
  b After drifting 200 m away from shore find the angle of elevation that the lighthouse is now at.

17 Two buildings 50 m and 75 m tall are separated by 70 m. Find the angle of elevation from the top of the shorter building to the top of the taller building.

18 Wayne says that the angle of elevation from A to B will be equal to the angle of depression from B to A. Is Wayne correct? Explain your answer.

INVESTIGATE: Calculation of heights

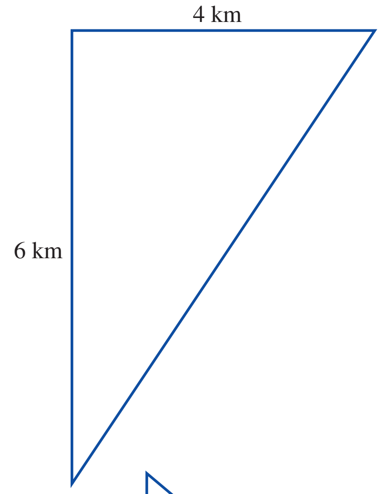
To measure the heights of trees and buildings around your school, try the following.

1 Measure your height at eye level.
2 Take a clinometer and from a point measure the angle of elevation to the top of the tree or building.
3 Measure your distance from the foot of the tree or building.
4 Use trigonometry to calculate the height, remembering to add your height at eye level to the result of the calculation.

Proportional diagrams

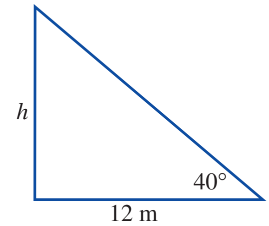
In many cases, we need only an approximate measurement for a practical problem. This type of answer can be obtained by drawing a scale diagram.

Consider the situation where a hiker walks 6 km due north, turns and walks 4 km due east. By drawing a diagram using a scale of 1 cm = 1 km, we can obtain an approximate measurement for the distance the hiker is from the starting point.



By measurement, we can see that the distance the hiker is from the starting point is approximately 7.2 km.

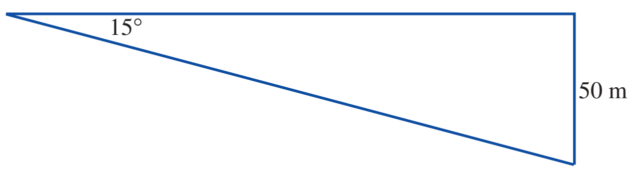
With a protractor, we can draw a scale diagram to solve problems involving angles. Suppose that the angle of elevation to the top of a tree is 40° from a point 12 m from the foot of the tree.



By measurement, the height of the tree is approximately 10 m.

In many situations a quick check of the accuracy of an answer is useful and can be made by using a scale drawing. In such cases the drawing would need to be only approximately to scale.

Suppose that you were told that the angle of depression from the top of a 50 m cliff to a ship out to sea was 15°. You were then told that this ship is 1 km from shore.



Using this diagram, we would estimate that the ship is only 190 m from shore. Such a diagram is a useful check to a calculation.

INVESTIGATE: Checking with a proportional diagram

Draw diagrams roughly to scale to check the results to the previous investigation.

Such diagrams are used to develop car rally courses, cross-country running courses and orienteering events.

INVESTIGATE: Using proportional diagrams

Plan a track for a cross-country run or orienteering event around your school.

1 Measure the length of each leg and the angle involved in each turn.

2 On a scale diagram, draw the course.

3 By measuring your diagram, calculate the approximate length of the course.

SUMMARY

Pythagoras' theorem

- When finding the hypotenuse of a right-angled triangle, use the formula:

$$c^2 = a^2 + b^2.$$

- To find a shorter side of a right-angled triangle use:

$$a^2 = c^2 - b^2 \text{ or } b^2 = c^2 - a^2.$$

Calculating trigonometric ratios

- $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$
- $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$
- $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$
- SOHCAHTOA — this acronym will help you remember trigonometric formulas.

Finding an unknown side

- Label the sides of the triangle opposite, adjacent and hypotenuse.
- Choose the correct ratio.
- Substitute given information.
- Make the unknown side the subject of the equation.
- Calculate.

Finding angles

- Label the sides of the triangle opposite, adjacent and hypotenuse.
- Choose the correct ratio.
- Substitute given information.
- Make the unknown angle the subject of the equation.
- Calculate by using the inverse trigonometric functions.

Angles of elevation and depression

- The angle of elevation is the angle we look up from the horizontal to see an object.
- The angle of depression is the angle we look down from the horizontal to see an object.
- Problems are solved using angles of elevation and depression by the same methods as for all right-angled triangles.

Proportional diagrams

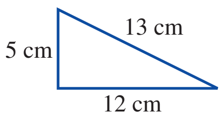
- A scale diagram can be drawn to obtain a reasonable estimate of a distance or angle.
- A diagram that is drawn roughly to scale can be used to check that an answer is reasonably accurate.

CHAPTER REVIEW

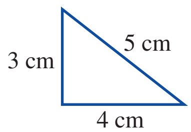
MULTIPLE CHOICE

1 MC Which of the triangles drawn below is not right angled?

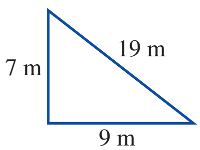
A



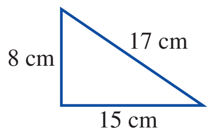
B



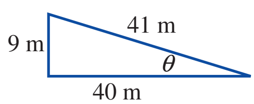
C



D



2 MC Look at the triangle below.



Statement 1. $\cos\theta = \frac{9}{41}$

Statement 2. $\tan\theta = \frac{9}{40}$

Which of the above statements is true?

A 1 only **B** 2 only
C both 1 and 2 **D** neither statement

3 MC Which of the following statements is correct?

A $\cos 30° = \tan 60°$ **B** $\cos 30° = \sin 60°$
C $\cos 30° = \sin 30°$ **D** $\cos 60° = \sin 60°$

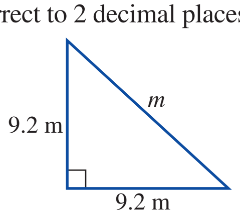
4 MC The exact value of $\cos\theta = \frac{\sqrt{3}}{2}$. The angle $\theta =$

A 30° **B** 45°
C 60° **D** 90°

SHORT ANSWER

1 Find the length of the side marked with a pronumeral, in each case writing your answer correct to 2 decimal places.

a



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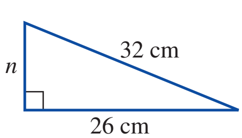

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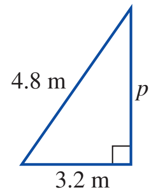

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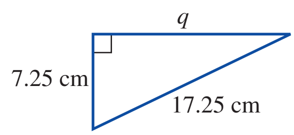
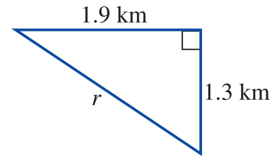
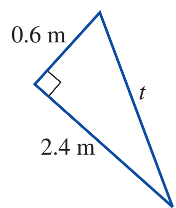


e



f



2 To travel between the towns of Bolong and Molong, you need to travel west along a road for 45 km, then north along another road for another 87 km. Calculate the straight-line distance between the two towns.

3 A rope is 80 m long and runs from a cliff top to the ground, 45 m from the base of the cliff. Calculate the height of the cliff, to the nearest metre.

4 Calculate each of the following, correct to 4 decimal places.

a $\sin 46°$ **b** $\tan 76°42'$
c $4.9 \cos 56°$ **d** $8.9 \sin 67°3'$
e $\dfrac{5.69}{\cos 75°}$ **f** $\dfrac{2.5}{\tan 9°42'}$

5 Calculate θ, correct to the nearest degree, given that:

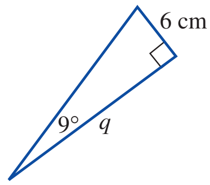
a $\cos\theta = 0.5874$ **b** $\tan\theta = 1.23$
c $\sin\theta = 0.8$.

6 Calculate θ, correct to the nearest minute, given that:

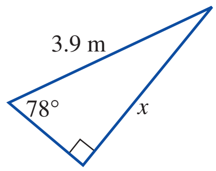
a $\cos\theta = 0.199$ **b** $\tan\theta = 0.5$
c $\sin\theta = 0.257$.

7 Find the length of each side marked with a pronumeral, correct to 1 decimal place.

a



b



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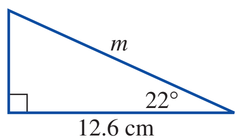

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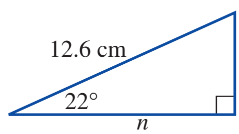
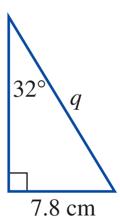
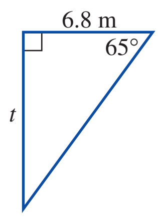


e



f



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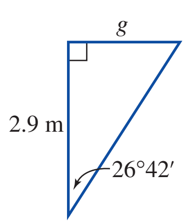

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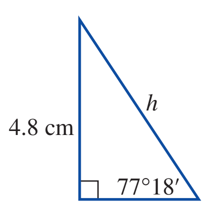

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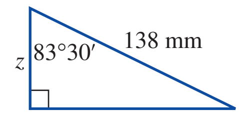

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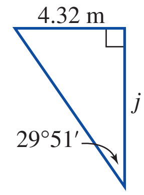

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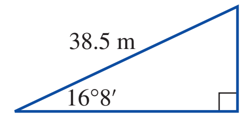
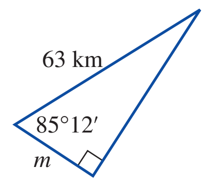


l



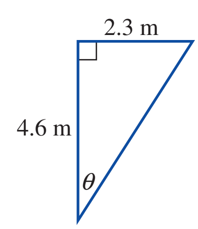
8 A rope that is used to support a flagpole makes an angle of 70° with the ground. If the rope is tied down 3.1 m from the foot of the flagpole, find the height of the flagpole, correct to 1 decimal place.

9 A dirt track runs off a road at an angle of 34° to the road. If I travel for 4.5 km along the dirt track, what is the shortest distance back to the road (correct to 1 decimal place)?

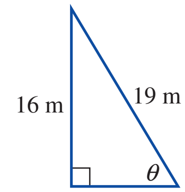
10 A fire is burning in a building and people need to be rescued. The fire brigade's ladder must reach a height of 60 m and must be angled at 70° to the horizontal. How long must the ladder be to complete the rescue?

11 Find the size of the angle marked θ in each of the following, giving your answer correct to the nearest degree.

a



b



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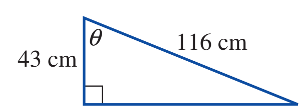
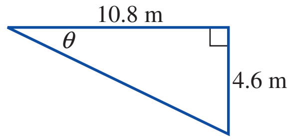
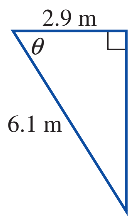


12 Find the size of the angle marked θ in each of the following, giving your answer correct to the nearest minute.

a



b



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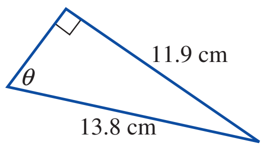
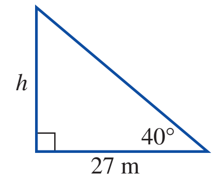


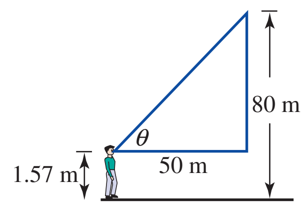
13 A kite on an 80 m string reaches a height of 50 m in a strong wind. Calculate the angle the string makes with the horizontal.

14 There is 50 m of line on a fishing reel. When all the line is out, the bait sits on the bed of a lake and has drifted 20 m from the boat. Calculate the angle that the fishing line makes with the vertical.

15 The top of a building is sighted at an angle of elevation of 40°, when an observer is 27 m back from the base. Calculate the height of the building, correct to the nearest metre.



16 Hakam stands 50 m back from the foot of an 80 m telephone tower. Hakam's eyes are at a height of 1.57 m. Calculate the angle of elevation that Hakam must look to see the top of the tower.

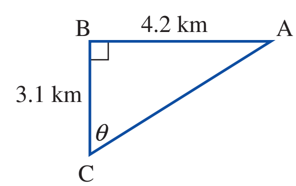


EXTENDED RESPONSE

1 On a bushwalk starting at point A, Sally walks 4.2 km due west to point B then turns due south for a distance of 3.1 km to point C.

a Calculate the distance, AC, that Sally must walk to return to her starting point.

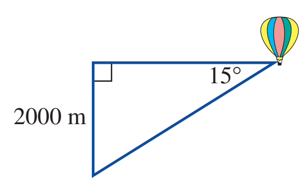
b Calculate the direction that Sally must walk, represented by the angle θ, correct to the nearest degree.



2 A hot-air balloon takes off and after 30 minutes of flying reaches an altitude of 2000 m. At that time, the angle of depression to its launch pad is 15°.

a Calculate the horizontal distance that the balloon has travelled in that half-hour (correct to the nearest 100 m).

b Calculate the angle of depression to the launch pad after the balloon has travelled 15 km in one direction (assuming that it maintains its altitude of 2000 m).



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Digital doc
Test Yourself
doc-1638
Chapter 13

eBookplus ACTIVITIES

Are you ready?

Digital docs *(page 414)*
- SkillSHEET 13.1 (doc-1621): Labelling sides of a right-angled triangle
- SkillSHEET 13.2 (doc-1622): Using Pythagoras' theorem
- SkillSHEET 13.3 (doc-1629): Rounding to a given number of decimal places
- SkillSHEET 13.4 (doc-1630): Solving equations of the type $a = \frac{x}{b}$ to find x
- SkillSHEET 13.5 (doc-1631): Solving equations of the type $a = \frac{b}{x}$ to find x
- SkillSHEET 13.7 (doc-1634): Rounding angles to the nearest degree

13A Pythagoras' theorem

Interactivity
- int-2406: Pythagoras *(page 416)*

Tutorial
- WE4 int-2336: Determine whether a triangle is right-angled. *(page 417)*

Digital docs
- SkillSHEET 13.1 (doc-1621): Labelling sides of a right-angled triangle *(page 418)*
- SkillSHEET 13.2 (doc-1622): Using Pythagoras' theorem *(page 418)*
- Spreadsheet (doc-1623): Pythagoras *(page 418)*
- GCprogram — Casio (doc-1624): Pythagoras *(page 419)*
- GCprogram — TI (doc-1625): Pythagoras *(page 419)*

13B Calculating trigonometric ratios

Digital docs
- Spreadsheet (doc-1626): Tangent *(page 421)*
- Spreadsheet (doc-1627): Sine *(page 423)*
- Spreadsheet (doc-1628): Cosine *(page 424)*
- SkillSHEET 13.3 (doc-1629): Rounding to a given number of decimal places *(page 426)*

13C Finding an unknown side

Interactivity
- int-2405: SOHCAHTOA *(page 429)*

Tutorial
- WE12 int-2337: Learn to find the length of an unknown side of a right-angled triangle. *(page 428)*
- WE14 int-2338: Learn to apply trigonometric ratios to a problem. *(page 429)*

Digital docs
- SkillSHEET 13.4 (doc-1630): Solving equations of the type $a = \frac{x}{b}$ to find x *(page 430)*
- SkillSHEET 13.5 (doc-1631): Solving equations of the type $a = \frac{b}{x}$ to find x *(page 431)*
- SkillSHEET 13.6 (doc-1632): Rearranging formulas *(page 431)*
- WorkSHEET 13.1 (doc-1633): Apply your knowledge of trigonometric ratios to finding side lengths. *(page 434)*

13D Finding angles

Tutorial
- WE15 int-2339: Learn to find the size of an unknown angle of a right-angled triangle. *(page 434)*

Digital docs
- SkillSHEET 13.7 (doc-1634): Rounding angles to the nearest degree *(page 437)*
- SkillSHEET 13.8 (doc-1635): Rounding angles to the nearest minute *(page 437)*
- SkillSHEET 13.9 (doc-1636): Rounding angles to the nearest second *(page 437)*

13E Angles of elevation and depression

Tutorial
- WE20 int-2340: Learn to apply knowledge of angles of depression. *(page 441)*

Digital docs
- WorkSHEET 13.2 (doc-1637): Apply your knowledge of trigonometry to problems. *(page 444)*

Chapter review

- Test yourself Chapter 13 (doc-1638): Take the end-of-chapter test to test your progress. *(page 449)*

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